Unit 5
Student Task Statements

Arithmetic in Base Ten

Click on a title in the list below to scroll directly to that lesson.

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Unit 5, Lesson 1

Using Decimals in a Shopping Context

Let's use what we know about decimals to make shopping decisions.

1.1 Snacks from the Concession Stand

Clare went to a concession stand that sells pretzels for $3.25, drinks for $1.85, and bags of popcorn for $0.99 each. She bought at least one of each item and spent no more than $10.

"Concession stand + spinning merry go round" by m01229 via Flickr. CC BY 2.0.

1. Could Clare have purchased 2 pretzels, 2 drinks, and 2 bags of popcorn? Explain your reasoning.

2. Could she have bought 1 pretzel, 1 drink, and 5 bags of popcorn? Explain your reasoning.

1.2 Planning a Dinner Party

You are planning a dinner party with a budget of $50 and a menu that consists of 1 main dish, 2 side dishes, and 1 dessert. There will be 8 guests at your party.

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Choose your menu items and decide on the quantities to buy so you stay on budget. If you choose meat, fish, or poultry for your main dish, plan to buy at least 0.5 pound per person.

Use the worksheet to record your choices and estimated costs. Then find the estimated total cost and cost per person. See examples in the first two rows.

1. The budget is $ ___________ per guest.

item	**quantity needed**	**advertised price**	**estimated subtotal (in dollars)**	**estimated cost per person (in dollars)**
ex. main dish: fish	4 pounds	$6.69 per pound	$4 \cdot 7 = 28$	$28 \div 8 = 3.50$
ex. dessert: cupcakes	8 cupcakes	$2.99 per 6 cupcakes	$2 \cdot 3 = 6$	$6 \div 8 = 0.75$
main dish:				
side dish 1:				
side dish 2:				
dessert:				
estimated total				

2. Is your estimated total close to your budget? If so, continue to the next question. If not, revise your menu choices until your estimated total is close to the budget.

3. Calculate the actual costs of the two most expensive items and add them. Show your reasoning.

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4. How will you know if your total cost for all menu items will or will not exceed your budget? Is there a way to predict this without adding all the exact costs? Explain your reasoning.

Are you ready for more?

How much would it cost to plant the grass on a football field? Explain or show your reasoning.

Lesson 1 Summary

We often use decimals when dealing with money. In these situations, sometimes we round and make estimates, and other times we calculate the numbers more precisely.

There are many different ways we can add, subtract, multiply, and divide decimals. When we perform these computations, it is helpful to understand the meanings of the digits in a number and the properties of operations. We will investigate how these understandings help us work with decimals in upcoming lessons.

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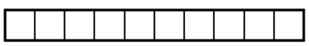
Unit 5, Lesson 2

Using Diagrams to Represent Addition and Subtraction

Let's represent addition and subtraction of decimals.

2.1 Changing Values

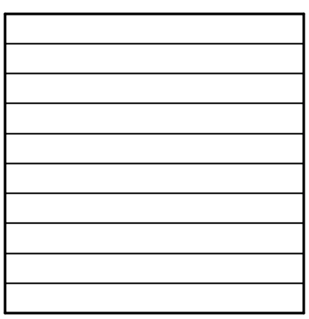
1. Here is a rectangle.



What number does the rectangle represent if each small square represents:

a. 1
b. 0.1
c. 0.01
d. 0.001

2. Here is a square.



What number does the square represent if each small rectangle represents:

a. 10

b. 0.1

c. 0.00001

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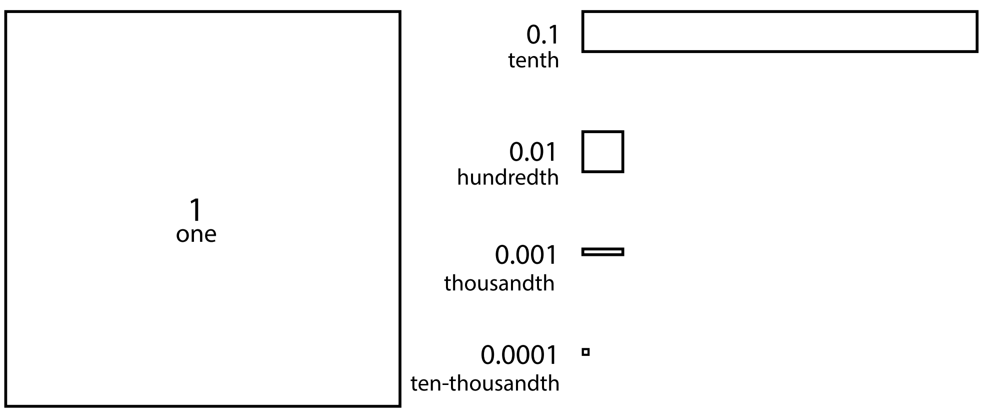
2.2 Squares and Rectangles

Interactive digital version available

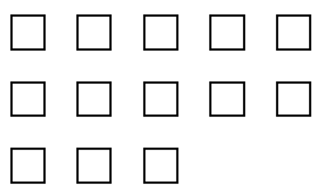
a.openup.org/ms-math/en/s/ccss-6-5-2-2

You may be familiar with base-ten blocks that represent ones, tens, and hundreds. Here are some diagrams that we will use to represent base-ten units.

- A large square represents 1 one.
- A medium rectangle represents 1 tenth.
- A medium square represents 1 hundredth.
- A small rectangle represents 1 thousandth.
- A small square represents 1 ten-thousandth.



1. Here is the diagram that Priya drew to represent 0.13. Draw a different diagram that represents 0.13. Explain why your diagram and Priya's diagram represent the same number.



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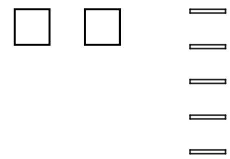
2. Here is the diagram that Han drew to represent 0.025. Draw a different diagram that represents 0.025. Explain why your diagram and Han's diagram represent the same number.



3. For each number, draw or describe two different diagrams that represent it.

   a. 0.1 b. 0.02 c. 0.004

4. Use diagrams of base-ten units to represent each sum. Think about how you could use as few units as possible to represent each number.

   a. $0.03 + 0.05$

   c. $0.4 + 0.7$

   b. $0.006 + 0.007$

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2.3 Finding Sums in Different Ways

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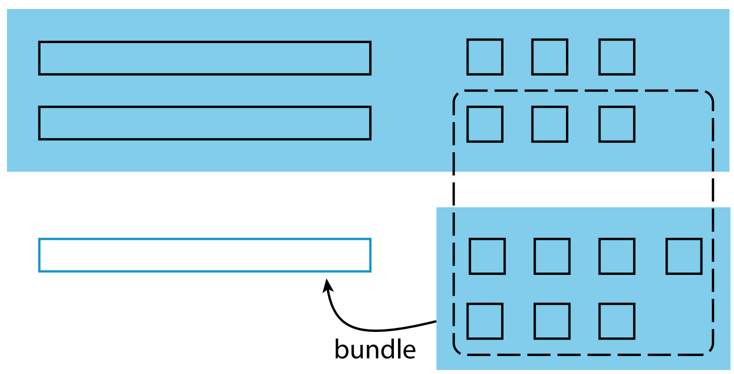
a.openup.org/ms-math/en/s/ccss-6-5-2-3

1. Here are two ways to calculate the value of $0.26 + 0.07$. In the diagram, each rectangle represents 0.1 and each square represents 0.01.

$$\begin{array}{r} \scriptstyle 1 \\ 0.26 \\ +\quad 0.07 \\ \hline 0.33 \end{array}$$



Use what you know about base-ten units and addition to explain:

a. Why ten squares can be “bundled” into a rectangle.

b. How this “bundling” is reflected in the computation.

2. Find the value of $0.38 + 0.69$ by drawing a diagram. Can you find the sum without bundling? Would it be useful to bundle some pieces? Explain your reasoning.

3. Calculate $0.38 + 0.69$. Check your calculation against your diagram in the previous question.

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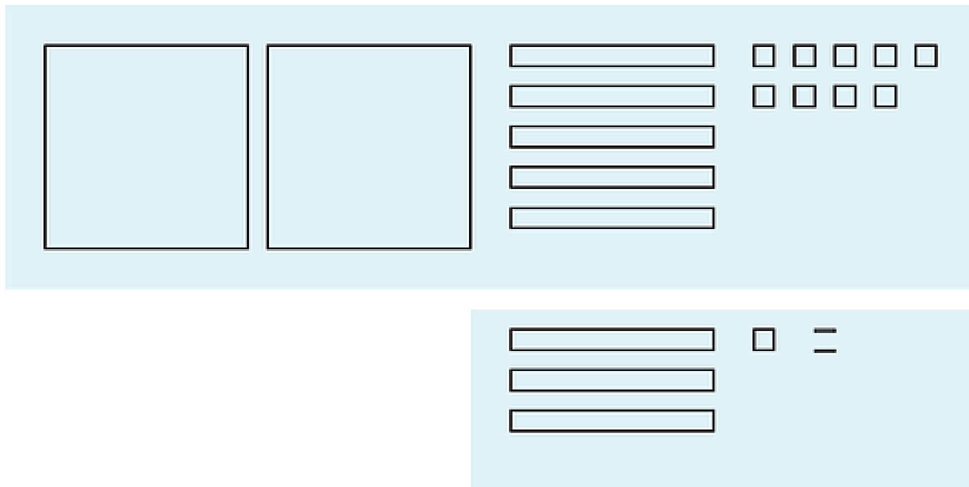
4. Find each sum. The larger square represents 1, the larger rectangle represents 0.1, the smaller square represents 0.01, and the smaller rectangle represents 0.001.



a.

b.

$$
\begin{array}{r}
6.03 \\
+\ 0.098 \\
\hline
\end{array}
$$

Are you ready for more?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

1. If you had 500 violet jewels and wanted to trade so that you carried as few jewels as possible, which jewels would you have?
2. Suppose you have 1 orange jewel, 2 yellow jewels, and 1 indigo jewel. If you're given 2 green jewels and 1 yellow jewels, what is the fewest number of jewels that could represent the value of the jewels you have?

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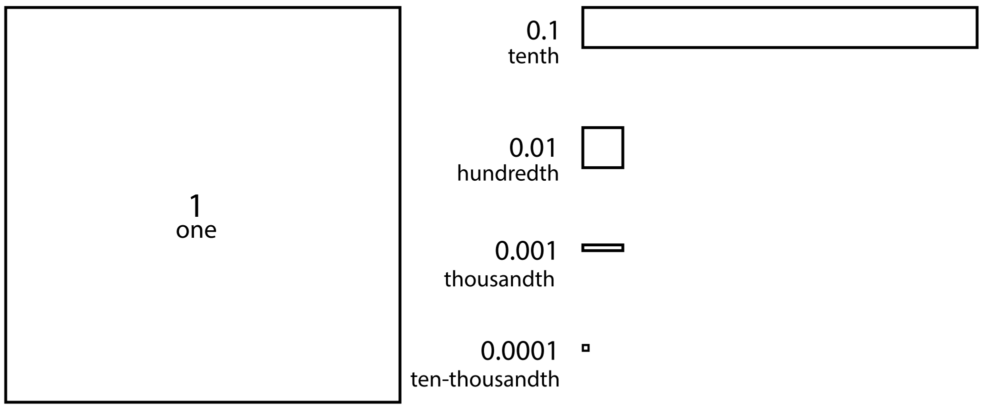
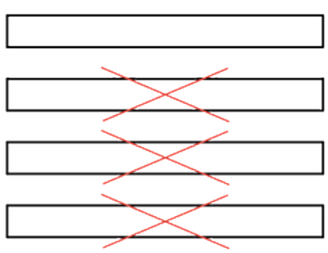
2.4 Representing Subtraction

Here are the diagrams you used to represent ones, tenths, hundredths, thousandths, and ten-thousandths.

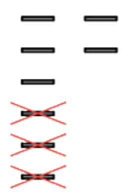


1. Here are diagrams that represent differences. Removed pieces are marked with Xs. For each diagram, write a numerical subtraction expression and determine the value of the expression.

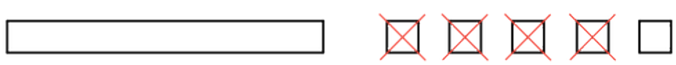
a.



b.



c.



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2. Express each subtraction in words.

   a. $0.05 - 0.02$

   b. $0.024 - 0.003$

   c. $1.26 - 0.14$

3. Find each difference by drawing a diagram and by calculating with numbers. Make sure the answers from both methods match. If not, check your diagram and your numerical calculation.

   a. $0.05 - 0.02$

   b. $0.024 - 0.003$

   c. $1.26 - 0.14$

Lesson 2 Summary

Base-ten diagrams represent collections of base-ten units—tens, ones, tenths, hundredths, etc. We can use them to help us understand sums of decimals.

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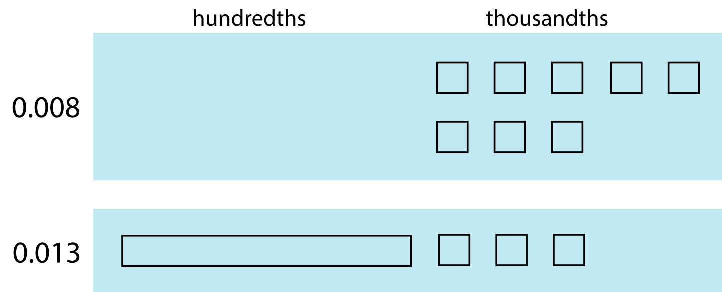
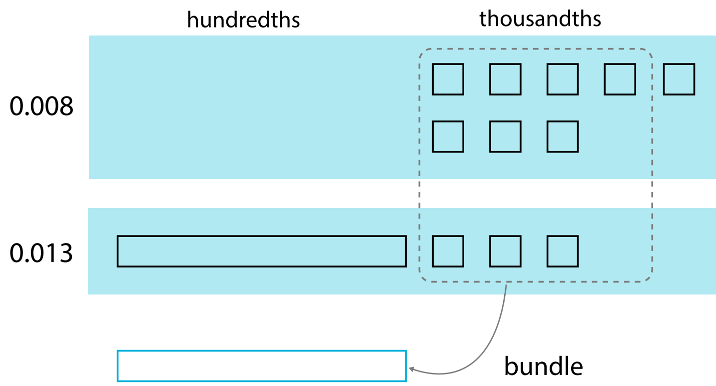
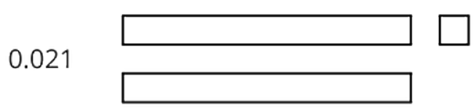
Here is a diagram of 0.008 and 0.013, where a square represents 0.001 and a rectangle (made up of ten squares) represents 0.01.



To find the sum, we can "bundle" (or compose) 10 thousandths as 1 hundredth.



Here is a diagram of the sum, which shows 2 hundredths and 1 thousandth.



We can use vertical calculation to find $0.008 + 0.013$. Notice that here 10 thousandths are also bundled (or composed) as 1 hundredth.

$$
\begin{array}{rcccc}
 & & & 1 & \\
 & 0. & 0 & 1 & 3 \\
+ & 0. & 0 & 0 & 8 \\
\hline
 & 0. & 0 & 2 & 1
\end{array}
$$

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Unit 5, Lesson 3

Adding and Subtracting Decimals with Few Non-Zero Digits

Let's add and subtract decimals.

3.1 Do the Zeros Matter?

1. Evaluate mentally: $1.009 + 0.391$

2. Decide if each equation is true or false. Be prepared to explain your reasoning.

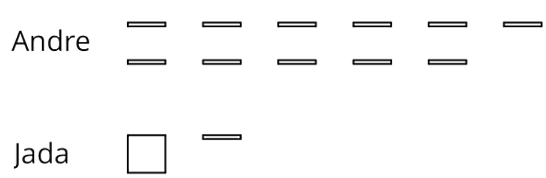
   a. $34.56000 = 34.56$ b. $25 = 25.0$ c. $2.405 = 2.45$

3.2 Calculating Sums

Interactive digital version available

a.openup.org/ms-math/en/s/ccss-6-5-3-2

1. Andre and Jada drew base-ten diagrams to represent $0.007 + 0.004$. Andre drew 11 small rectangles. Jada drew only two figures: a square and a small rectangle.



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a. If both students represented the sum correctly, what value does each small rectangle represent? What value does each square represent?

b. Draw or describe a diagram that could represent the sum $0.008 + 0.07$.

2. Here are two calculations of $0.2 + 0.05$. Which is correct? Explain why one is correct and the other is incorrect.

$$\begin{array}{r} 0.2 \\ +\ 0.05 \\ \hline 0.25 \end{array} \qquad \begin{array}{r} 0.2 \\ +\ 0.05 \\ \hline 0.07 \end{array}$$

3. Compute each sum. If you get stuck, draw base-ten diagrams to help you.

a. $\begin{array}{r} 0.11 \\ +\ 0.005 \\ \hline \end{array}$

b. $0.209 + 0.01$

c. $10.2 + 1.1456$

3.3 Subtracting Decimals of Different Lengths

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To represent $0.4 - 0.03$, Diego and Noah drew different diagrams. Each rectangle represented 0.1. Each square represented 0.01.

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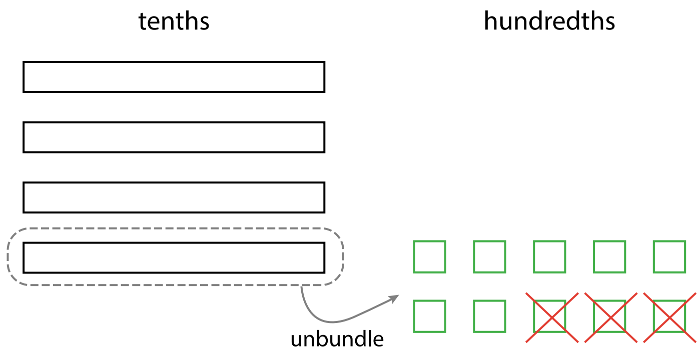
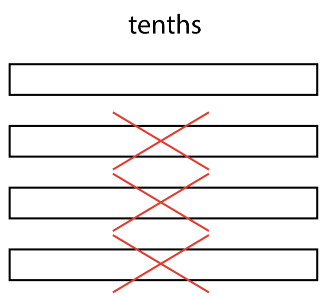
- Diego started by drawing 4 rectangles to represent 0.4. He then replaced 1 rectangle with 10 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 3 rectangles and 7 squares in his diagram.



Diego's Method

- Noah started by drawing 4 rectangles to represent 0.4. He then crossed out 3 rectangles to represent the subtraction, leaving 1 rectangle in his diagram.



Noah's Method

1. Do you agree that either diagram correctly represents $0.4 - 0.03$? Discuss your reasoning with a partner.

2. To represent $0.4 - 0.03$, Elena drew another diagram. She also started by drawing 4 rectangles. She then replaced all 4 rectangles with 40 squares and crossed out 3 squares to represent subtraction of 0.03, leaving 37 squares in her diagram. Is her diagram correct? Discuss your reasoning with a partner.

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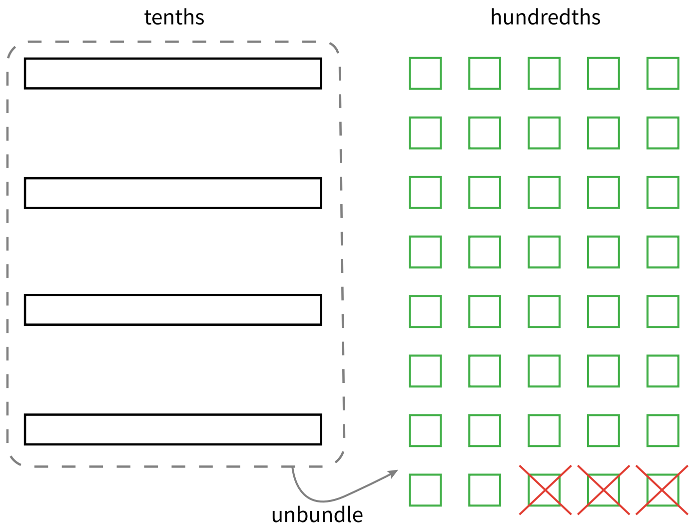

Elena's Method

3. Find each difference. Explain or show your reasoning.

   a. $0.3 - 0.05$

   c. $1.03 - 0.06$

   b. $2.1 - 0.4$

   d. $0.02 - 0.007$

Are you ready for more?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in

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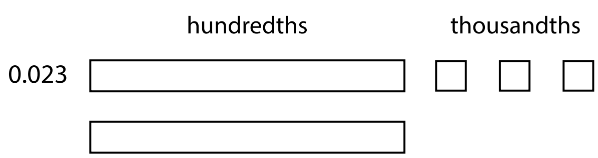
the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

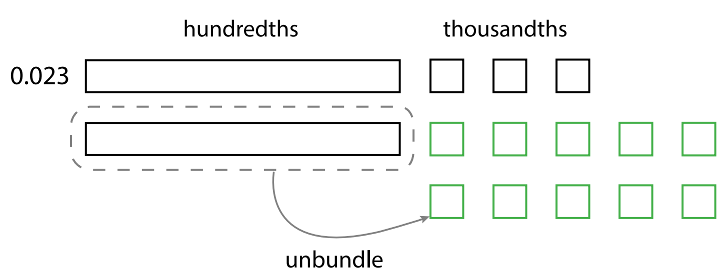
At the Auld Shoppe, a shopper buys items that are worth 2 yellow jewels, 2 green jewels, 2 blue jewels, and 1 indigo jewel. If they came into the store with 1 red jewel, 1 yellow jewel, 2 green jewels, 1 blue jewel, and 2 violet jewels, what jewels do they leave with? Assume the shopkeeper gives them their change using as few jewels as possible.

Lesson 3 Summary

Base-ten diagrams can help us understand subtraction as well as addition. Suppose we are finding $0.023 - 0.007$. Here is a diagram showing 0.023, or 2 hundredths and 3 thousandths.



Subtracting 7 thousandths means removing 7 small squares, but we do not have enough to remove. Because 1 hundredth is equal to 10 thousandths, we can “unbundle” (or decompose) one of the hundredths (1 rectangle) into 10 thousandths (10 small squares).



We now have 1 hundredth and 13 thousandths, from which we can remove 7 thousandths.

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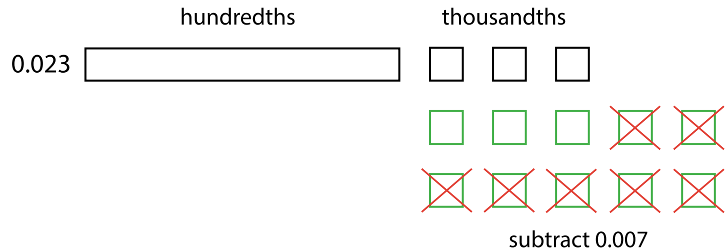
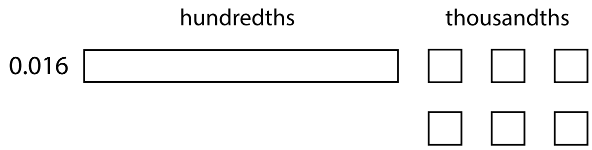


We have 1 hundredth and 6 thousandths remaining, so $0.023 - 0.007 = 0.016$.



Here is a vertical calculation of $0.023 - 0.007$.

$$
\begin{array}{rcccc}
 & & & 1 & 10 \\
 & 0. & 0 & \cancel{2} & \cancel{3} \\
- & 0. & 0 & 0 & 7 \\
\hline
 & 0. & 0 & 1 & 6
\end{array}
$$

In both calculations, notice that a hundredth is unbundled (or decomposed) into 10 thousandths in order to subtract 7 thousandths.

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Unit 5, Lesson 4

Adding and Subtracting Decimals with Many Non-Zero Digits

Let's practice adding and subtracting decimals.

4.1 The Cost of a Photo Print

1. Here are three ways to write a subtraction calculation. What do you notice? What do you wonder?

$$\begin{array}{r} 5 \\ -\ 0.17 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -\ 0.17 \\ \hline \end{array} \qquad \begin{array}{r} 5 \\ -\ 0.17 \\ \hline \end{array}$$

2. Clare bought a photo for 17 cents and paid with a $5 bill. Look at the previous question. Which way of writing the numbers could Clare use to find the change she should receive? Be prepared to explain how you know.

3. Find the amount of change that Clare should receive. Show your reasoning, and be prepared to explain how you calculate the difference of 0.17 and 5.

4.2 Decimals All Around

1. Find the value of each expression. Show your reasoning.

   a. $11.3 - 9.5$ b. $318.8 - 94.63$ c. $0.02 - 0.0116$

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2. Discuss with a partner:
    - Which method or methods did you use in the previous question? Why?
    - In what ways were your methods effective? Was there an expression for which your methods did not work as well as expected?

3. Lin's grandmother ordered needles that were 0.3125 inches long to administer her medication, but the pharmacist sent her needles that were 0.6875 inches long. How much longer were these needles than the ones she ordered? Show your reasoning.

4. There is 0.162 liter of water in a 1-liter bottle. How much more water should be put in the bottle so it contains exactly 1 liter? Show your reasoning.

5. One micrometer is 1 millionth of a meter. A red blood cell is about 7.5 micrometers in diameter. A coarse grain of sand is about 70 micrometers in diameter. Find the difference between the two diameters in *meters*. Show your reasoning.

4.3 Missing Numbers

Write the missing digits in each calculation so that the value of each sum or difference is correct. Be prepared to explain your reasoning.

1.

$$\begin{array}{r} 0.404 \\ +\ \square\square\square\square \\ \hline 1 \end{array}$$

2.

$$\begin{array}{r} 9.8765 \\ +\ \square\square\square\square\square\square \\ \hline 10 \end{array}$$

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3.

$$\begin{array}{r} 0.7 \\ - \ \boxed{}\boxed{}\boxed{}\boxed{} \\ \hline 0.012 \end{array}$$

4.

$$\begin{array}{r} 7 \\ - \ \boxed{}\boxed{}\boxed{}\boxed{}\boxed{} \\ \hline 3.4567 \end{array}$$

5.

$$\begin{array}{r} 70 \\ - \ \boxed{}\boxed{}\boxed{}\boxed{}\boxed{}\boxed{} \\ \hline 0.0089 \end{array}$$

Are you ready for more?

In a cryptarithmetic puzzle, the digits 0-9 are represented using the first 10 letters of the alphabet. Use your understanding of decimal addition to determine which digits go with the letters A, B, C, D, E, F, G, H, I, and J. How many possibilities can you find?

$$\begin{array}{r} \text{I H F . I J} \\ + \ \text{J I I . F I} \\ \hline \text{E J I . I E} \end{array}$$

Lesson 4 Summary

Base-ten diagrams work best for representing subtraction of numbers with few non-zero digits, such as $0.16 - 0.09$. For numbers with many non-zero digits, such as $0.25103 - 0.04671$, it would take a long time to draw the base-ten diagram. With vertical calculations, we can find this difference efficiently.

Thinking about base-ten diagrams can help us make sense of this calculation.

$$\begin{array}{rcccccc} & & & & \scriptstyle 10 & & \\ & & & \scriptstyle 4 & \scriptstyle \cancel{0} & \scriptstyle 10 & \\ & 0. & 2 & \cancel{5} & \cancel{1} & \cancel{0} & 3 \\ - & 0. & 0 & 4 & 6 & 7 & 1 \\ \hline & 0. & 2 & 0 & 4 & 3 & 2 \end{array}$$

The thousandth in 0.25103 is unbundled (or decomposed) to make 10 ten-thousandths so that we can subtract 7 ten-thousandths. Similarly, one of the hundredths in 0.25103 is unbundled (or decomposed) to make 10 thousandths.

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Unit 5, Lesson 5
Decimal Points in Products

Let's look at products that are decimals.

5.1 Multiplying by 10

1. In which equation is the value of x the largest?

   $x \cdot 10 = 810$ $\quad$ $x \cdot 10 = 81$ $\quad$ $x \cdot 10 = 8.1$ $\quad$ $x \cdot 10 = 0.81$

2. How many times the size of 0.81 is 810?

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5.2 Fractionally Speaking: Powers of Ten

Work with a partner to answer the following questions. One person should answer the questions labeled "Partner A," and the other should answer those labeled "Partner B." Then compare the results.

1. Find each product or quotient. Be prepared to explain your reasoning.

Partner A	Partner B
a. $250 \cdot \frac{1}{10}$	a. $250 \div 10$
b. $250 \cdot \frac{1}{100}$	b. $250 \div 100$
c. $48 \div 10$	c. $48 \cdot \frac{1}{10}$
d. $48 \div 100$	d. $48 \cdot \frac{1}{100}$

2. Use your work in the previous problems to find $720 \cdot (0.1)$ and $720 \cdot (0.01)$. Explain your reasoning.

Pause here for a class discussion.

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3. Find each product. Show your reasoning.

   a. $36 \cdot (0.1)$

   b. $(24.5) \cdot (0.1)$

   c. $(1.8) \cdot (0.1)$

   d. $54 \cdot (0.01)$

   e. $(9.2) \cdot (0.01)$

4. Jada says: "If you multiply a number by 0.001, the decimal point of the number moves three places to the left." Do you agree with her statement? Explain your reasoning.

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5.3 Fractionally Speaking: Multiples of Powers of Ten

1. Select **all** expressions that are equivalent to $(0.6) \cdot (0.5)$. Be prepared to explain your reasoning.

   a. $6 \cdot (0.1) \cdot 5 \cdot (0.1)$

   b. $6 \cdot (0.01) \cdot 5 \cdot (0.1)$

   c. $6 \cdot \frac{1}{10} \cdot 5 \cdot \frac{1}{10}$

   d. $6 \cdot \frac{1}{1{,}000} \cdot 5 \cdot \frac{1}{100}$

   e. $6 \cdot (0.001) \cdot 5 \cdot (0.01)$

   f. $6 \cdot 5 \cdot \frac{1}{10} \cdot \frac{1}{10}$

   g. $\frac{6}{10} \cdot \frac{5}{10}$

2. Find the value of $(0.6) \cdot (0.5)$. Show your reasoning.

3. Find the value of each product by writing and reasoning with an equivalent expression with fractions.

   a. $(0.3) \cdot (0.02)$

   b. $(0.7) \cdot (0.05)$

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Are you ready for more?

Ancient Romans used the letter I for 1, V for 5, X for 10, L for 50, C for 100, D for 500, and M for 1,000.

Write a problem involving merchants at an agora, an open-air market, that uses multiplication of numbers written with Roman numerals.

Lesson 5 Summary

We can use fractions like $\frac{1}{10}$ and $\frac{1}{100}$ to reason about the location of the decimal point in a product of two decimals.

Let’s take $24 \cdot (0.1)$ as an example. There are several ways to find the product:

- We can interpret it as 24 groups of 1 tenth (or 24 tenths), which is 2.4.
- We can think of it as $24 \cdot \frac{1}{10}$, which is equal to $\frac{24}{10}$ (and also equal to 2.4).
- Multiplying by $\frac{1}{10}$ has the same result as dividing by 10, so we can also think of the product as $24 \div 10$, which is equal to 2.4.

Similarly, we can think of $(0.7) \cdot (0.09)$ as 7 tenths times 9 hundredths, and write:

$$\left(7 \cdot \frac{1}{10}\right) \cdot \left(9 \cdot \frac{1}{100}\right)$$

We can rearrange whole numbers and fractions:

$$(7 \cdot 9) \cdot \left(\frac{1}{10} \cdot \frac{1}{100}\right) = 63 \cdot \frac{1}{1,000} = \frac{63}{1,000}$$

This tells us that $(0.7) \cdot (0.09) = 0.063$.

Here is another example: To find $(1.5) \cdot (0.43)$, we can think of 1.5 as 15 tenths and 0.43 as 43 hundredths. We can write the tenths and hundredths as fractions and rearrange the factors.

$$\left(15 \cdot \frac{1}{10}\right) \cdot \left(43 \cdot \frac{1}{100}\right) = 15 \cdot 43 \cdot \frac{1}{1,000}$$

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Multiplying 15 and 43 gives us 645, and multiplying $\frac{1}{10}$ and $\frac{1}{100}$ gives us $\frac{1}{1,000}$. So $(1.5) \cdot (0.43)$ is $645 \cdot \frac{1}{1,000}$, which is 0.645.

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Unit 5, Lesson 6
Methods for Multiplying Decimals

Let’s look at some ways we can represent multiplication of decimals.

6.1 Which One Doesn’t Belong: Products

Which expression doesn’t belong? Explain your reasoning.

A. $2 \cdot (0.3)$ B. $2 \cdot 3 \cdot (0.1)$ C. $6 \cdot (0.1)$ D. $(0.1) \cdot 6$

6.2 Using Properties of Numbers to Reason about Multiplication

1. Elena and Noah used different methods to compute $(0.23) \cdot (1.5)$. Both computations were correct.

Elena's Method	Noah's Method
$(0.23) \cdot 100 = 23$	$0.23 = \frac{23}{100}$
$(1.5) \cdot 10 = 15$	$1.5 = \frac{15}{10}$
$23 \cdot 15 = 345$	$\frac{23}{100} \cdot \frac{15}{10} \quad \frac{345}{1{,}000}$
$345 \div 1{,}000 = 0.345$	$\frac{345}{1{,}000} = 0.345$

Analyze the two methods, then discuss these questions with your partner.

- Which method makes more sense to you? Why?

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- What might Elena do to compute $(0.16) \cdot (0.03)$? What might Noah do to compute $(0.16) \cdot (0.03)$? Will the two methods result in the same value?

2. Compute each product using the equation $21 \cdot 47 = 987$ and what you know about fractions, decimals, and place value. Explain or show your reasoning.

   a. $(2.1) \cdot (4.7)$

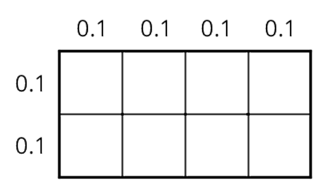
   b. $21 \cdot (0.047)$

   c. $(0.021) \cdot (4.7)$

6.3 Using Area Diagrams to Reason about Multiplication

1. In the diagram, the side length of each square is 0.1 unit.

   a. Explain why the area of each square is *not* 0.1 square unit.



   b. How can you use the area of each square to find the area of the rectangle? Explain or show your reasoning.

   c. Explain how the diagram shows that the equation $(0.4) \cdot (0.2) = 0.08$ is true.

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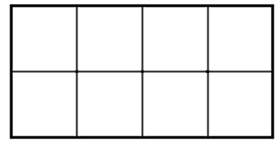
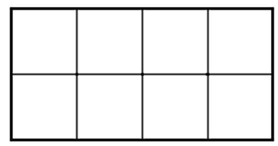
2. Label the squares with their side lengths so the area of this rectangle represents $40 \cdot 20$.



   a. What is the area of each square?

   b. Use the squares to help you find $40 \cdot 20$. Explain or show your reasoning.

3. Label the squares with their side lengths so the area of this rectangle represents $(0.04) \cdot (0.02)$.
   Next, use the diagram to help you find $(0.04) \cdot (0.02)$.
   Explain or show your reasoning.



Lesson 6 Summary

Here are three other ways to calculate a product of two decimals such as $(0.04) \cdot (0.07)$.

- First, we can multiply each decimal by the same power of 10 to obtain whole-number factors.

  $$(0.04) \cdot 100 = 4$$
  $$(0.07) \cdot 100 = 7$$
  $$4 \cdot 7 = 28$$

  Because we multiplied both 0.04 and 0.07 by 100 to get 4 and 7, the product 28 is $(100 \cdot 100)$ times the original product, so we need to divide 28 by 10,000.

$$28 \div 10{,}000 = 0.0028$$

- Second, we can write each decimal as a fraction, $0.04 = \frac{4}{100}$ and $0.07 = \frac{7}{100}$, and multiply them.

$$\frac{4}{100} \cdot \frac{7}{100} = \frac{28}{10{,}000} = 0.0028$$

- Third, we can use an area model. The product $(0.04) \cdot (0.07)$ can be thought of as the area of a rectangle with side lengths of 0.04 unit and 0.07 unit.

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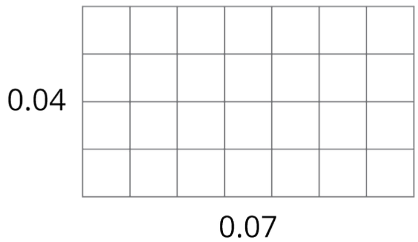

In this diagram, each small square is 0.01 unit by 0.01 unit. Its area, in square units, is therefore $\left(\frac{1}{100} \cdot \frac{1}{100}\right)$, which is $\frac{1}{10{,}000}$.

Because the rectangle is composed of 28 small squares, its area, in square units, must be:

$$28 \cdot \frac{1}{10,000} = \frac{28}{10,000} = 0.0028$$

All three calculations show that $(0.04) \cdot (0.07) = 0.0028$.

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Unit 5, Lesson 7

Using Diagrams to Represent Multiplication

Let's use area diagrams to find products.

7.1 Estimate the Product

For each of the following products, choose the best estimate of its value. Be prepared to explain your reasoning.

1. $(6.8) \cdot (2.3)$
    - 1.40
    - 14
    - 140
2. $74 \cdot (8.1)$
    - 5.6
    - 56
    - 560
3. $166 \cdot (0.09)$
    - 1.66
    - 16.6
    - 166

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4. $(3.4) \cdot (1.9)$
    - 6.5
    - 65
    - 650

7.2 Connecting Area Diagrams to Calculations with Whole Numbers

Interactive digital version available

a.openup.org/ms-math/en/s/ccss-6-5-7-2

1. Here are three ways of finding the area of a rectangle that is 24 units by 13 units.

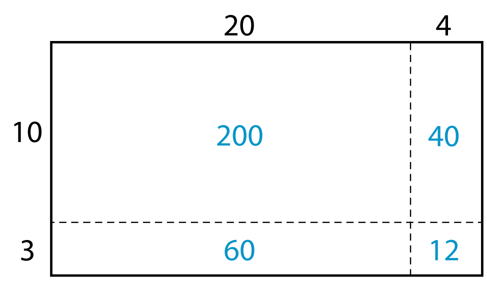

Diagram 1

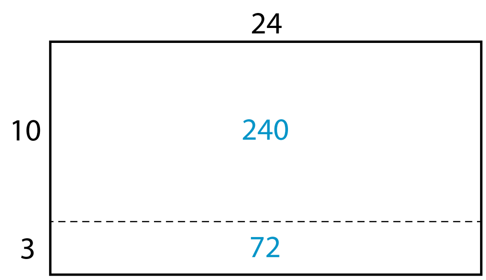

Diagram 2

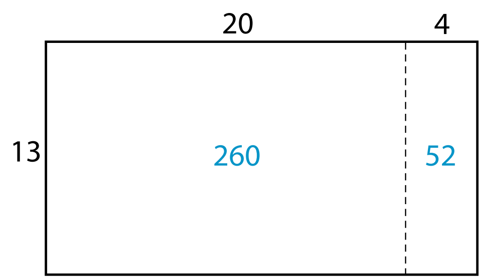

Diagram 3

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Discuss with your partner:

a. What do the diagrams have in common? How are they alike?

b. How are they different?

c. If you were to find the area of a rectangle that is 37 units by 19 units, which of the three ways of decomposing the rectangle would you use? Why?

2. You may be familiar with different ways to write multiplication calculations. Here are two ways to calculate 24 times 13.

$$\begin{array}{r} 2\ 4 \\ \times \quad 1\ 3 \\ \hline \left.\begin{array}{r} 1\ 2 \\ 6\ 0 \\ 4\ 0 \\ +\quad 2\ 0\ 0 \end{array}\right\}\text{partial products} \\ \hline 3\ 1\ 2 \end{array}$$

Calculation A

$$\begin{array}{r} 2\ 4 \\ \times \quad 1\ 3 \\ \hline 7\ 2 \\ +\quad 2\ 4\ 0 \\ \hline 3\ 1\ 2 \end{array}$$

Calculation B

Discuss with your partner:

a. In Calculation A, how are each of the partial products obtained? For instance, where does the 12 come from?

b. In Calculation B, how are the 72 and 240 obtained?

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c. Look at the diagrams in the first question. Which diagram corresponds to Calculation A? Which one corresponds to Calculation B?

d. How are the partial products in Calculation A and the 72 and 240 in Calculation B related to the numbers in the diagrams?

3. Use the two following methods to find the product of 18 and 14, then compare the values obtained.

a. Calculate numerically.

$$\begin{array}{r} 1\ 8 \\ \times \quad 1\ 4 \\ \hline \end{array}$$

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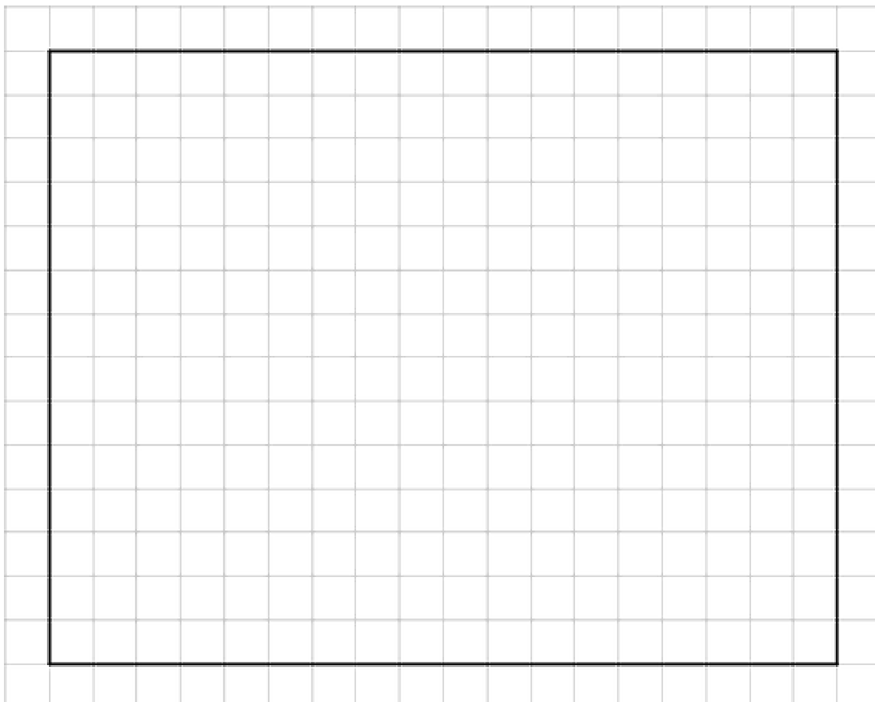
b. Here is a rectangle that is 18 units by 14 units. Find its area, in square units by decomposing it. Show your reasoning.



4. Compare the values of $18 \cdot 14$ that you obtained using the two methods. If they are not the same, check your work.

7.3 Connecting Area Diagrams to Calculations with Decimals

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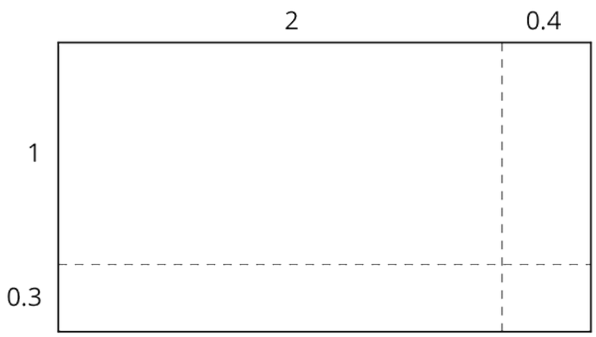
1. You can use area diagrams to represent products of decimals. Here is an area diagram that represents $(2.4) \cdot (1.3)$.



   a. Find the region that represents $(0.4) \cdot (0.3)$. Label it with its area of 0.12.

   b. Label each of the other regions with their respective areas.

   c. Find the value of $(2.4) \cdot (1.3)$. Show your reasoning.

2. Here are two ways of calculating $(2.4) \cdot (1.3)$. Analyze the calculations and discuss the following questions with a partner.

$$\begin{array}{lr} & 2.4 \\ \times & 1.3 \\ \hline & 0.12 \\ & 0.6 \\ & 0.4 \\ + & 2 \\ \hline & 3.12 \end{array} \left.\vphantom{\begin{array}{c}0\\0\\0\\0\end{array}}\right\} \text{partial products}$$

Calculation A

$$\begin{array}{lr} & 2.4 \\ \times & 1.3 \\ \hline & 0.72 \\ + & 2.4 \\ \hline & 3.12 \end{array}$$

Calculation B

   a. In Calculation A, where does the 0.12 and other partial products come from? In Calculation B, where do the 0.72 and 2.4 come from?

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b. In each calculation, why are the numbers below the horizontal line aligned vertically the way they are?

3. Find the product of $(3.1) \cdot (1.5)$ by drawing and labeling an area diagram. Show your reasoning.

4. Show how to calculate $(3.1) \cdot (1.5)$ using numbers without a diagram. Be prepared to explain your reasoning. If you are stuck, use the examples in a previous question to help you.

Are you ready for more?

How many hectares is the property of your school? How many morgens is that?

7.4 Using the Partial Products Method

1. Label the area diagram to represent $(2.5) \cdot (1.2)$ and to find that product.

   a. Decompose each number into its base-ten units (ones, tenths, etc.) and write them in the boxes on each side of the rectangle.

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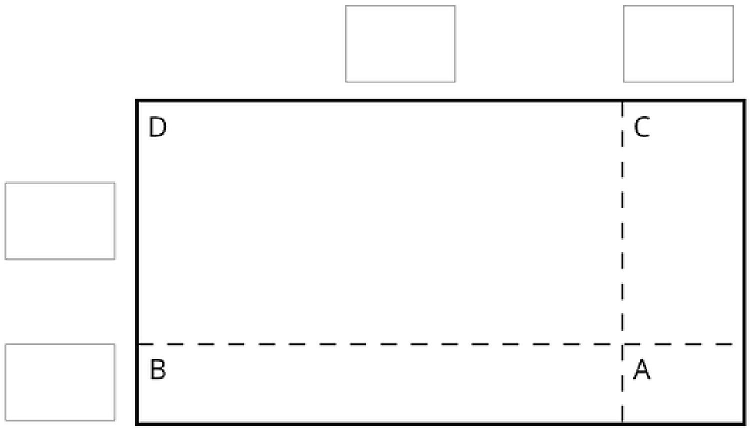

b. Label Regions A, B, C, and D with their areas. Show your reasoning.

c. Find the product that the area diagram represents. Show your reasoning.

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2. Here are two ways to calculate $(2.5) \cdot (1.2)$. Each number with a box gives the area of one or more regions in the area diagram.

Calculation A		Calculation B	
2.5		2.5	
× 1.2		× 1.2	
0.1	$\boxed{}$	0.5	$\boxed{}$
0.4	$\boxed{}$	+ 2.5	$\boxed{}$
0.5	$\boxed{}$	3.00	
+ 2.0	$\boxed{}$		
3.00			

Calculation A Calculation B

a. In the boxes next to each number, write the letter(s) of the corresponding region(s).

b. In Calculation B, which two numbers are being multiplied to obtain 0.5? Which two are being multiplied to obtain 2.5?

Lesson 7 Summary

Suppose that we want to calculate the product of two numbers that are written in base ten. To explain how, we can use what we know about base-ten numbers and areas of rectangles.

Here is a diagram of a rectangle whose side lengths are 3.4 units and 1.2 units. Its area, in square units, is the product $(3.4) \cdot (1.2)$. To calculate this product and find the area of the rectangle, we can

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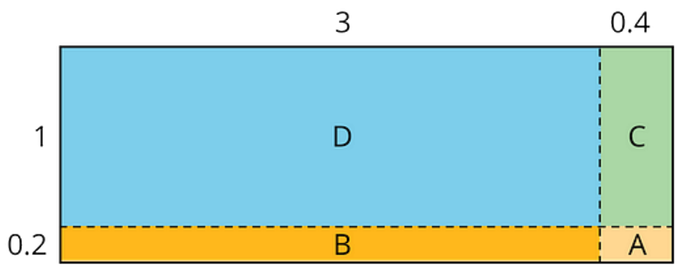
decompose each side length into its base-ten units, $3.4 = 3 + 0.4$ and $1.2 = 1 + 0.2$
, decomposing the rectangle into four smaller sub-rectangles.



We can rewrite the product and expand it twice:

$$\begin{aligned}(3.4) \cdot (1.2) &= (3 + 0.4) \cdot (1 + 0.2) \\ &= (3 + 0.4) \cdot 1 + (3 + 0.4) \cdot 0.2 \\ &= 3 \cdot 1 + 3 \cdot (0.2) + (0.4) \cdot 1 + (0.4) \cdot (0.2)\end{aligned}$$

In the last expression, each of the four terms is called a partial product. Each partial product gives the area of a sub-rectangle in the diagram. The sum of the four partial products gives the area of the entire rectangle.

We can show the horizontal calculations above as two vertical calculations.

$$\begin{array}{rrl} & 3.4 & \\ \times & 1.2 & \\ \hline & \overset{1}{0}.08 & \text{A} \\ & 0.6 & \text{B} \\ & 0.4 & \text{C} \\ + & 3 & \text{D} \\ \hline & 4.08 & \end{array} \qquad \begin{array}{rrl} & 3.4 & \\ \times & 1.2 & \\ \hline & \overset{1}{0}.68 & \text{A} + \text{B} \\ + & 3.4 & \text{C} + \text{D} \\ \hline & 4.08 & \end{array}$$

The vertical calculation on the left is an example of the partial products method. It shows the values of each partial product and the letter of the corresponding sub-rectangle. Each partial product gives an area:

- A is 0.2 unit by 0.4 unit, so its area is 0.08 square unit.
- B is 3 unit by 0.2 unit, so its area is 0.6 square unit.
- C is 0.4 unit by 1 unit, so its area is 0.4 square unit.
- D is 3 units by 1 unit, so its area is 3 square units.

- The sum of the partial products is $0.08 + 0.6 + 0.4 + 3$, so the area of the rectangle is 4.08 square units.

The calculation on the right shows the values of two products. Each value gives a combined area of two sub-rectangles:

- The combined regions of A and B have an area of 0.68 square units; 0.68 is the value of $(3 + 0.4) \cdot 0.2$.
- The combined regions of C and D have an area of 3.4 square units; 3.4 is the value of $(3 + 0.4) \cdot 1$.
- The sum of the values of two products is $0.68 + 3.4$, so the area of the rectangle is 4.08 square units.

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Unit 5, Lesson 8
Calculating Products of Decimals

Let’s multiply decimals.

8.1 Number Talk: Twenty Times a Number

Evaluate mentally.

$20 \cdot 5$

$20 \cdot (0.8)$

$20 \cdot (0.04)$

$20 \cdot (5.84)$

8.2 Calculating Products of Decimals

1. A common way to find a product of decimals is to calculate a product of whole numbers, then place the decimal point in the product.

$$
\begin{array}{rr}
 & 25 \\
\times & 12 \\
\hline
 & 50 \\
+ & 250 \\
\hline
 & 300
\end{array}
$$

$25 \cdot 12 = 300$

$(2.5) \cdot (1.2) = 3.00$

Here is an example for $(2.5) \cdot (1.2)$.

Use what you know about decimals and place value to explain why the decimal point of the product is placed where it is.

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2. Use the method shown in the first question to calculate each product.

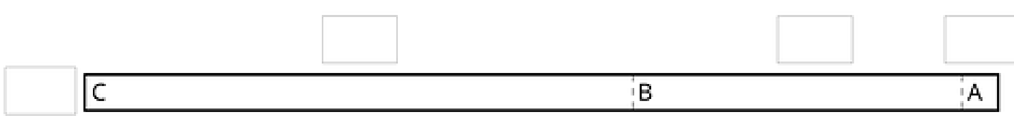
   a. $(4.6) \cdot (0.9)$

   b. $(16.5) \cdot (0.7)$

3. Use area diagrams to check your earlier calculations. For each problem:
   - Decompose each number into its base-ten units and write them in the boxes on each side of the rectangle.
   - Write the area of each lettered region in the diagram. Then find the area of the entire rectangle. Show your reasoning.

   a. $(4.6) \cdot (0.9)$

   b. $(16.5) \cdot (0.7)$



4. About how many centimeters are in 6.25 inches if 1 inch is about 2.5 centimeters? Show your reasoning.

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8.3 Practicing Multiplication of Decimals

1. Calculate each product. Show your reasoning. If you get stuck, draw an area diagram to help.

   a. $(5.6) \cdot (1.8)$

   b. $(0.008) \cdot (7.2)$

2. A rectangular playground is 18.2 meters by 12.75 meters.

   a. Find its area in square meters. Show your reasoning.

   b. If 1 meter is approximately 3.28 feet, what are the approximate side lengths of the playground in feet? Show your reasoning.

Are you ready for more?

1. Write the following expressions as decimals.

   a. $1 - 0.1$

   b. $1 - 0.1 + 10 - 0.01$

   c. $1 - 0.1 + 10 - 0.01 + 100 - 0.001$

2. Describe the decimal that results as this process continues.

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3. What would happen to the decimal if all of the positive and negative signs became multiplication symbols? Explain your reasoning.

Lesson 8 Summary

We can use $84 \cdot 43$ and what we know about place value to find $(8.4) \cdot (4.3)$.

Since 8.4 is 84 tenths and 4.3 is 43 tenths, then:

$$(8.4) \cdot (4.3) = \frac{84}{10} \cdot \frac{43}{10}$$
$$(8.4) \cdot (4.3) = \frac{84 \cdot 43}{100}$$

That means we can compute $84 \cdot 43$ and then divide by 100 to find $(8.4) \cdot (4.3)$.

$$84 \cdot 43 = 3612$$
$$(8.4) \cdot (4.3) = 36.12$$

Using fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ allows us to find the product of two decimals using the following steps:

- Write each decimal factor as a product of a whole number and a fraction.
- Multiply the whole numbers.
- Multiply the fractions.
- Multiply the products of the whole numbers and fractions.

We know multiplying by fractions such as $\frac{1}{10}$, $\frac{1}{100}$, and $\frac{1}{1,000}$ is the same as dividing by 10, 100, and 1,000, respectively. This means we can move the decimal point in the whole-number product to the left the appropriate number of spaces to correctly place the decimal point.

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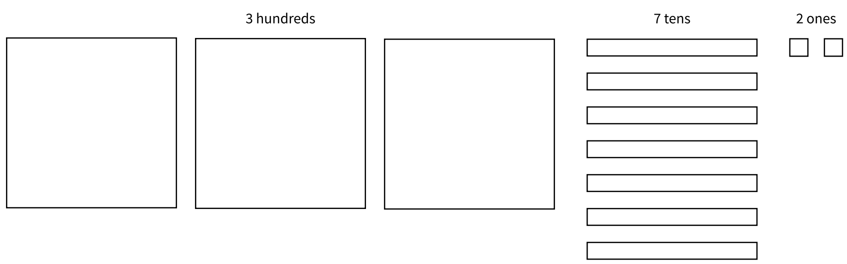
Unit 5, Lesson 9
Using the Partial Quotients Method

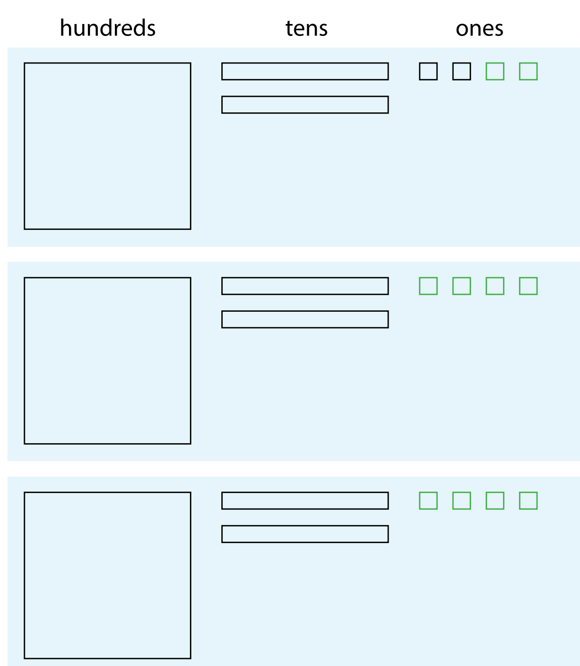
Let's divide whole numbers.

9.1 Using Base-Ten Diagrams to Calculate Quotients

Elena used base-ten diagrams to find $372 \div 3$. She started by representing 372.



She made 3 groups, each with 1 hundred. Then, she put the tens and ones in each of the 3 groups. Here is her diagram for $372 \div 3$.



Discuss with a partner:

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- Elena’s diagram for 372 has 7 tens. The one for $372 \div 3$ has only 6 tens. Why?
- Where did the extra ones (small squares) come from?

9.2 Using the Partial Quotients Method to Calculate Quotients

1. Andre calculated $657 \div 3$ using a method that was different from Elena’s.

He started by writing the dividend (657) and the divisor (3).

$$\begin{array}{r} 3\,\overline{)\,657} \end{array}$$

He then subtracted 3 groups of different amounts from 657, starting with 3 groups of 200…

$$\begin{array}{r} 200 \\ 3\,\overline{)\,657} \\ -\,600 \\ \hline 57 \end{array}$$

…then 3 groups of 10, and then 3 groups of 9.

$$\begin{array}{r} 9 \\ 10 \\ 200 \\ 3\,\overline{)\,657} \\ -\,600 \\ \hline 57 \\ -\,30 \\ \hline 27 \\ -\,27 \\ \hline 0 \end{array}$$

André calculated 200 + 10 + 9 and then wrote 219.

$$\begin{array}{r} \boxed{219} \\ 9 \\ 10 \\ 200 \\ 3\,\overline{)\,657} \\ -\,600 \\ \hline 57 \\ -\,30 \\ \hline 27 \\ -\,27 \\ \hline 0 \end{array}$$

Discuss the following questions with a partner:

- Andre subtracted 600 from 657. What does the 600 represent?
- Andre wrote 10 above the 200, and then subtracted 30 from 57. How is the 30 related to the 10?
- What do the numbers 200, 10, and 9 represent?
- What is the meaning of the 0 at the bottom of Andre’s work?

2. How might Andre calculate $896 \div 4$? Explain or show your reasoning.

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9.3 What’s the Quotient?

1. Find the quotient of $1,332 \div 9$ using one of the methods you have seen so far. Show your reasoning.

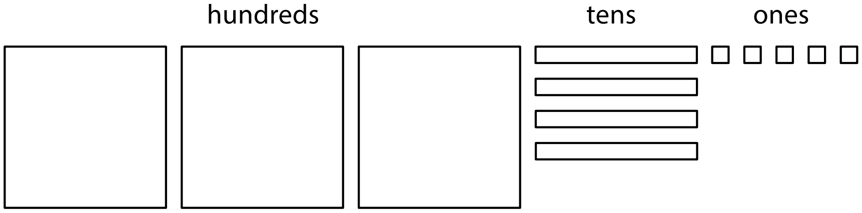
2. Find each quotient and show your reasoning. Use the partial quotients method at least once.

   a. $1,115 \div 5$
   b. $665 \div 7$
   c. $432 \div 16$

Lesson 9 Summary

We can find the quotient $345 \div 3$ in different ways.

One way is to use a base-ten diagram to represent the hundreds, tens, and ones and to create equal-sized groups.



We can think of the division by 3 as splitting up 345 into 3 equal groups.

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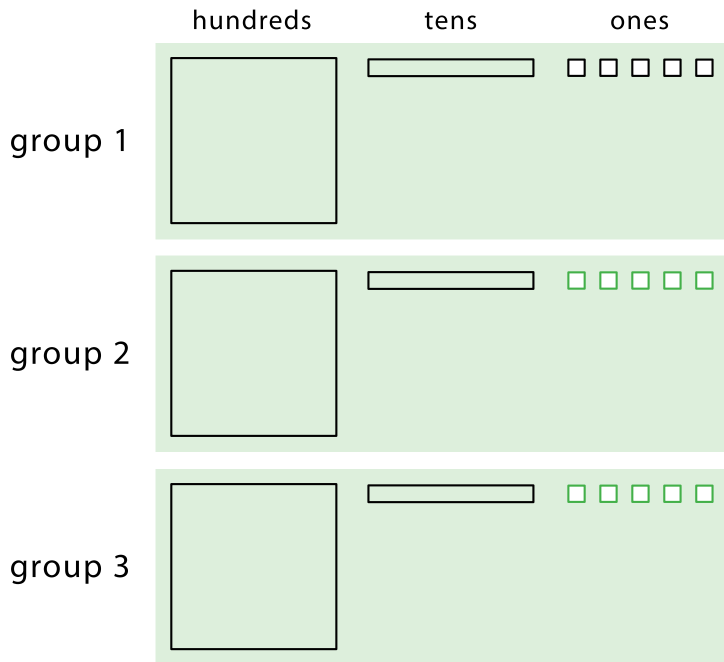

Each group has 1 hundred, 1 ten, and 5 ones, so $345 \div 3 = 115$. Notice that in order to split 345 into 3 equal groups, one of the tens had to be unbundled or decomposed into 10 ones.

Another way to divide 345 by 3 is by using the partial quotients method, in which we keep subtracting 3 groups of some amount from 345.

$$\begin{array}{rl} \boxed{1\ 1\ 5} & \\ 5 & \\ 1\ 0 & \\ 1\ 0\ 0 & \\ 3\,\overline{)\,3\ 4\ 5} & \\ -\ 3\ 0\ 0 & \longleftarrow \text{3 groups of 100} \\ \hline 4\ 5 & \\ -\ 3\ 0 & \longleftarrow \text{3 groups of 10} \\ \hline 1\ 5 & \\ -\ 1\ 5 & \longleftarrow \text{3 groups of 5} \\ \hline 0 & \end{array}
\qquad
\begin{array}{rl} \boxed{1\ 1\ 5} & \\ 5\ 0 & \\ 5\ 0 & \\ 1\ 5 & \\ 3\,\overline{)\,3\ 4\ 5} & \\ -\ 4\ 5 & \longleftarrow \text{3 groups of 15} \\ \hline 3\ 0\ 0 & \\ -1\ 5\ 0 & \longleftarrow \text{3 groups of 50} \\ \hline 1\ 5\ 0 & \\ -1\ 5\ 0 & \longleftarrow \text{3 groups of 5 0} \\ \hline 0 & \end{array}$$

- In the calculation on the left, first we subtract 3 groups of 100, then 3 groups of 10, and then 3 groups of 5. Adding up the partial quotients $(100 + 10 + 5)$ gives us 115.

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- The calculation on the right shows a different amount per group subtracted each time (3 groups of 15, 3 groups of 50, and 3 more groups of 50), but the total amount in each of the 3 groups is still 115. There are other ways of calculating $345 \div 3$ using the partial quotients method.

Both the base-ten diagrams and partial quotients methods are effective. If, however, the dividend and divisor are large, as in $1{,}248 \div 26$, then the base-ten diagrams will be time-consuming.

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Unit 5, Lesson 10

Using Long Division

Let’s use long division.

10.1 Number Talk: Estimating Quotients

Estimate these quotients mentally.

$500 \div 7$

$1{,}394 \div 9$

10.2 Lin Uses Long Division

Lin has a method of calculating quotients that is different from Elena’s method and Andre’s method. Here is how she found the quotient of $657 \div 3$:

Lin arranged the numbers for vertical calculations.

Her plan was to divide each digit of 657 into 3 groups, starting with the 6 hundreds.

$$3\,\overline{)\,6\;5\;7}$$

There are 3 groups of 2 in 6, so Lin wrote 2 at the top and subtracted 6 from the 6, leaving 0.

Then, she brought down the 5 tens of 657.

$$\begin{array}{r} 2 \\ \overline{)\,6\;5\;7} \\ -\,6 \\ \hline 0\;5 \end{array}$$

There are 3 groups of 1 in 5, so she wrote 1 at the top and subtracted 3 from 5, which left a remainder of 2.

$$\begin{array}{r} 2\;1 \\ \overline{)\,6\;5\;7} \\ -\,6 \\ \hline 5 \\ -\,3 \\ \hline 2 \end{array}$$

She brought down the 7 ones of 657 and wrote it next to the 2, which made 27.

There are 3 groups of 9 in 27, so she wrote 9 at the top and subtracted 27, leaving 0.

$$\begin{array}{r} 2\;1\;9 \\ \overline{)\,6\;5\;7} \\ -\,6 \\ \hline 5 \\ -\,3 \\ \hline 2\;7 \\ -\,2\;7 \\ \hline 0 \end{array}$$

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1. Discuss with your partner how Lin's method is similar to and different from drawing base-ten diagrams or using the partial quotients method.
   - Lin subtracted $3 \cdot 2$, then $3 \cdot 1$, and lastly $3 \cdot 9$. Earlier, Andre subtracted $3 \cdot 200$, then $3 \cdot 10$, and lastly $3 \cdot 9$. Why did they have the same quotient?
   - In the third step, why do you think Lin wrote the 7 next to the remainder of 2 rather than adding 7 and 2 to get 9?

2. Lin's method is called **long division**. Use this method to find the following quotients. Check your answer by multiplying it by the divisor.

   a. $846 \div 3$ b. $1{,}816 \div 4$ c. $768 \div 12$

10.3 Dividing Whole Numbers

1. Find each quotient.

   a. $633 \div 3$ b. $1001 \div 7$

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2. Here is Priya's calculation of $906 \div 3$.

$$\begin{array}{r} 3\;2\;0 \\ 3\,\overline{)\,9\;0\;6} \\ -\,9 \\ \hline 0\;6 \\ -6 \\ \hline 0 \end{array}$$

a. Priya wrote 320 for the value of $906 \div 3$. Check her answer by multiplying it by 3. What product do you get and what does it tell you about Priya's answer?

b. Describe Priya's mistake, then show the correct calculation and answer.

Lesson 10 Summary

Long division is another method for calculating quotients. It relies on place value to perform and record the division.

When we use long division, we work from left to right and with one digit at a time, starting with the leftmost digit of the dividend. We remove the largest group possible each time, using the placement of the digit to indicate the size of each group. Here is an example of how to find $948 \div 3$ using long division.

$$\begin{array}{rl} 3\;1\;6 & \\ 3\,\overline{)\,9\;4\;8} & \\ -\,9 & \longleftarrow \text{3 groups of 3 (hundreds)} \\ \hline 4 & \\ -\,3 & \longleftarrow \text{3 groups of 1 (ten)} \\ \hline 1\;8 & \\ -\,1\;8 & \longleftarrow \text{3 groups of 6 (ones)} \\ \hline 0 & \end{array}$$

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- We start by dividing 9 hundreds into 3 groups, which means 3 hundreds in each group. Instead of writing 300, we simply write 3 in the hundreds place, knowing that it means 3 hundreds.
- There are no remaining hundreds, so we work with the tens. We can make 3 groups of 1 ten in 4 tens, so we write 1 in the tens place above the 4 of 948. Subtracting 3 tens from 4 tens, we have a remainder of 1 ten.
- We know that 1 ten is 10 ones. Combining these with the 8 ones from 948, we have 18 ones. We can make 3 groups of 6, so we write 6 in the ones place.

In total, there are 3 groups of 3 hundreds, 1 ten, and 6 ones in 948, so $948 \div 3 = 316$.

Glossary Terms

long division

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Unit 5, Lesson 11
Dividing Numbers that Result in Decimals

Let’s find quotients that are not whole numbers.

11.1 Number Talk: Evaluating Quotients

Find the quotients mentally.

$400 \div 8$

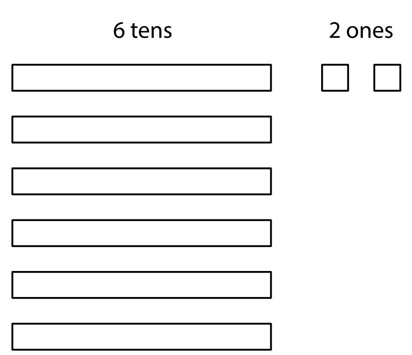
$80 \div 8$

$16 \div 8$

$496 \div 8$

11.2 Keep Dividing

Here is how Mai used base-ten diagrams to calculate $62 \div 5$.



She started by representing 62.

She then made 5 groups, each with 1 ten. There was 1 ten left. She unbundled it into 10 ones and distributed the ones across the 5 groups.

Here is her diagram for $62 \div 5$.

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1. Discuss these questions with a partner and write down your answers:

   a. Mai should have a total of 12 ones, but her diagram shows only 10. Why?

   b. She did not originally have tenths, but in her diagram each group has 4 tenths. Why?

   c. What value has Mai found for $62 \div 5$? Explain your reasoning.

2. Find the quotient of $511 \div 5$ by drawing base-ten diagrams or by using the partial quotients method. Show your reasoning. If you get stuck, work with your partner to find a solution.

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3. Four students share a \$271 prize from a science competition. How much does each student get if the prize is shared equally? Show your reasoning.

11.3 Using Long Division to Calculate Quotients

1. Here is how Lin calculated $62 \div 5$.

Lin set up the numbers for long division.	She subtracted 5 times 1 from the 6, which leaves a remainder of 1. She wrote the 2 from 62 next to the 1, which made 12, and subtracted 5 times 2 from 12.	Lin drew a vertical line and a decimal point, separating the ones and tenths place. 12 – 10 is 2. She wrote 0 to the right of the 2, which made 20.	Lastly, she subtracted 5 times 4 from 20, which left no remainder. At the top she wrote 4 next to the decimal point.
$5\overline{)6\ 2}$	$\begin{array}{r} 1 \\ 5\overline{)6\ 2} \\ -\ 5 \\ \hline 1\ 2 \\ -\ 1\ 0 \\ \hline 2 \end{array}$	$\begin{array}{r\|l} 1\ 2 & . \\ 5\overline{)6\ 2} & \\ -\ 5 & \\ \hline 1\ 2 & \\ -\ 1\ 0 & \\ \hline 2 & 0 \end{array}$	$\begin{array}{r\|l} 1\ 2 & .\ 4 \\ 5\overline{)6\ 2} & \\ -\ 5 & \\ \hline 1\ 2 & \\ -\ 1\ 0 & \\ \hline 2 & 0 \\ -\ 2 & 0 \\ \hline & 0 \end{array}$

Discuss with your partner:

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- Lin put a 0 after the remainder of 2. Why? Why does this 0 not change the value of the quotient?
- Lin subtracted 5 groups of 4 from 20. What value does the 4 in the quotient represent?
- What value did Lin find for $62 \div 5$?

2. Use long division to find the value of each expression. Then pause so your teacher can review your work.

   a. $126 \div 8$

   b. $90 \div 12$

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3. Use long division to show that:

   a. $5 \div 4$, or $\frac{5}{4}$, is 1.25.

   b. $4 \div 5$, or $\frac{4}{5}$, is 0.8.

   c. $1 \div 8$, or $\frac{1}{8}$, is 0.125.

   d. $1 \div 25$, or $\frac{1}{25}$, is 0.04.

4. Noah said we cannot use long division to calculate $10 \div 3$ because there will always be a remainder.

   a. What do you think Noah meant by “there will always be a remainder”?

   b. Do you agree with his statement? Why or why not?

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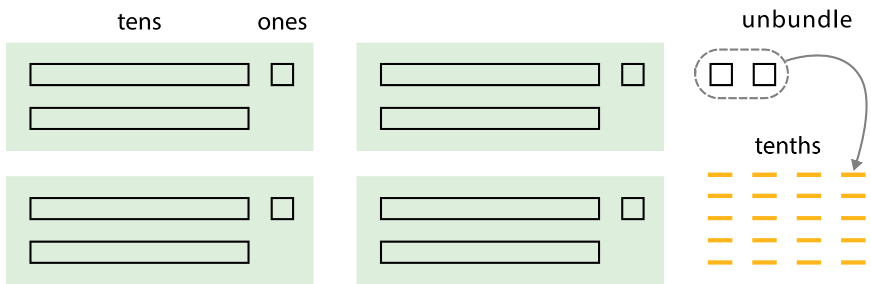
Lesson 11 Summary

Dividing a whole number by another whole number does not always produce a whole-number quotient. Let's look at $86 \div 4$, which we can think of as dividing 86 into 4 equal groups.



We can see in the base-ten diagram that there are 4 groups of 21 in 86 with 2 ones left over. To find the quotient, we need to distribute the 2 ones into the 4 groups. To do this, we can unbundle or decompose the 2 ones into 20 tenths, which enables us to put 5 tenths in each group.

Once the 20 tenths are distributed, each group will have 2 tens, 1 one, and 5 tenths, so $86 \div 4 = 21.5$.

$$
\begin{array}{r|l}
21. & 5 \\
4\overline{)86} & \\
-8 & \\
\hline
6 & \\
-4 & \\
\hline
2 & 0 \\
-2 & 0 \\
\hline
 & 0
\end{array}
$$

We can also calculate $86 \div 4$ using long division.

The calculation shows that, after removing 4 groups of 21, there are 2 ones remaining. We can continue dividing by writing a 0 to the right of the 2 and thinking of that remainder as 20 tenths, which can then be divided into 4 groups.

To show that the quotient we are working with now is in the tenth place, we put a decimal point to the right of the 1 (which is in the ones place) at the top. It may also be helpful to draw a vertical line to separate the ones and the tenths.

There are 4 groups of 5 tenths in 20 tenths, so we write 5 in the tenths place at the top. The calculation likewise shows $86 \div 4 = 21.5$.

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Unit 5, Lesson 12

Dividing Decimals by Whole Numbers

Let's divide decimals by whole numbers.

12.1 Number Talk: Dividing by 4

Find each quotient mentally.

$80 \div 4$

$12 \div 4$

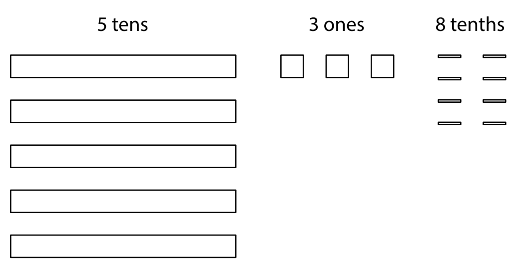
$1.2 \div 4$

$81.2 \div 4$

12.2 Using Diagrams to Represent Division

To find $53.8 \div 4$ using diagrams, Elena began by representing 53.8.

She placed 1 ten into each group, unbundled the remaining 1 ten into 10 ones, and went on distributing the units.



This diagram shows Elena's initial placement of the units and the unbundling of 1 ten.

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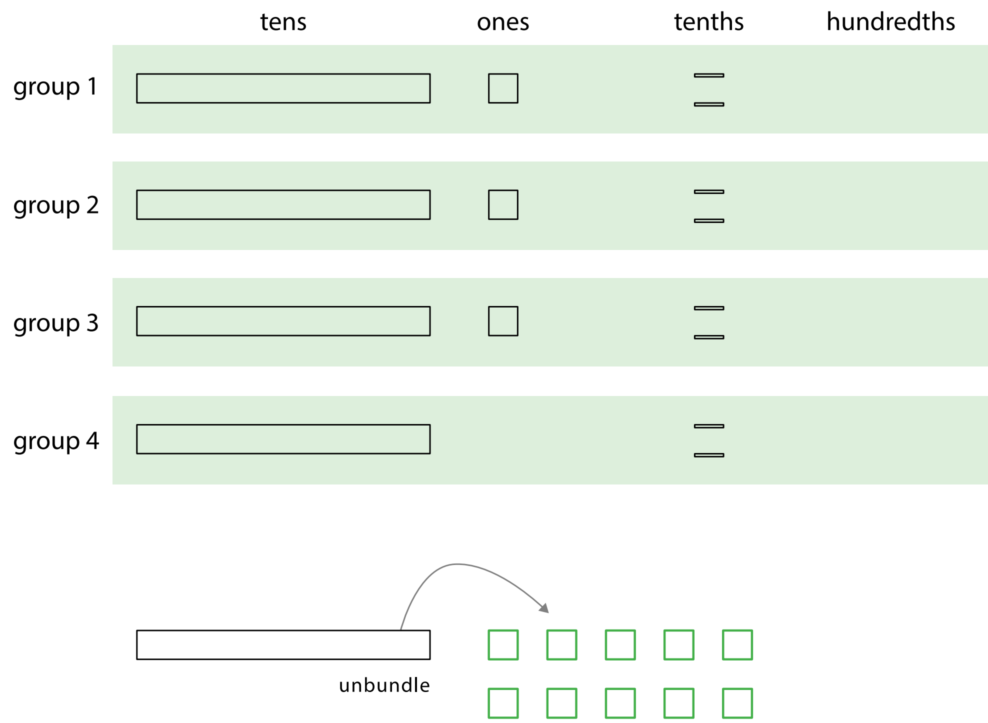

1. Complete the diagram by continuing the division process. How would you use the available units to make 4 equal groups?

   As the units get placed into groups, show them accordingly and cross out those pieces from the bottom. If you unbundle a unit, draw the resulting pieces.

2. What value did you find for $53.8 \div 4$? Be prepared to explain your reasoning.

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3. Use long division to find $53.8 \div 4$. Check your answer by multiplying it by the divisor 4.

4. Use long division to find $77.4 \div 5$. If you get stuck, you can draw diagrams or use another method.

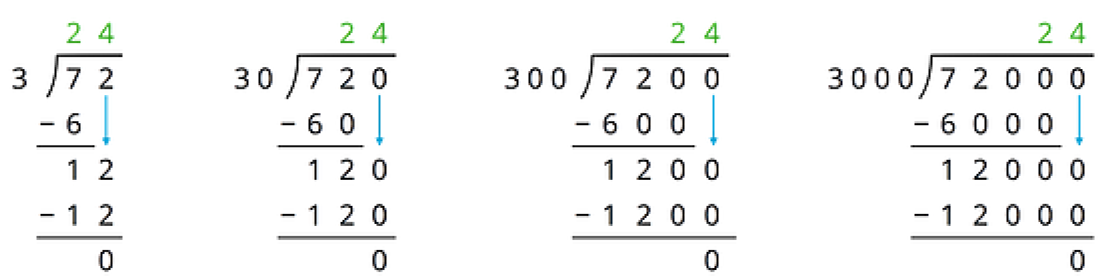
Are you ready for more?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

A group of 4 craftsmen are paid 1 of each jewel. If they split the jewels evenly amongst themselves, which jewels does each craftsman get?

12.3 Dividends and Divisors

Analyze the dividends, divisors, and quotients in the calculations, then answer the questions.



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1. Complete each sentence. In the calculations above:

   - Each dividend is ______ times the dividend to the left of it.
   - Each divisor is ______ times the divisor to the left of it.
   - Each quotient is _____________________ the quotient to the left of it.

2. Suppose we are writing a calculation to the right of $72{,}000 \div 3{,}000$. Which expression has a quotient of 24? Be prepared to explain your reasoning.

   a. $72{,}000 \div 30{,}000$

   b. $720{,}000 \div 300{,}000$

   c. $720{,}000 \div 30{,}000$

   d. $720{,}000 \div 3{,}000$

3. Suppose we are writing a calculation to the left of $72 \div 3$. Write an expression that would also give a quotient of 24. Be prepared to explain your reasoning.

4. Decide which of the following expressions would have the same value as $250 \div 10$. Be prepared to share your reasoning.

   a. $250 \div 0.1$

   b. $25 \div 1$

   c. $2.5 \div 1$

   d. $2.5 \div 0.1$

   e. $2,500 \div 100$

   f. $0.25 \div 0.01$

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Lesson 12 Summary

We know that fractions such as $\frac{6}{4}$ and $\frac{60}{40}$ are equivalent because:

- Both the numerator and denominator of $\frac{60}{40}$ have a factor of 10, so it can be written as $\frac{6}{4}$.
- Both fractions can be simplified to $\frac{3}{2}$.
- 600 divided by 400 is 1.5, and 60 divided by 40 is also 1.5.

Just like fractions, division expressions can be equivalent. For example, the expressions $540 \div 90$ and $5{,}400 \div 900$ are both equivalent to $54 \div 9$ because:

- They all have a quotient of 6.
- The dividend and the divisor in $540 \div 90$ are each 10 times the dividend and divisor in $54 \div 9$. Those in $5{,}400 \div 900$ are each 100 times the dividend and divisor in $54 \div 90$. In both cases, the quotient does not change.

This means that an expression such as $5.4 \div 0.9$ also has the same value as $54 \div 9$. Both the dividend and divisor of $5.4 \div 0.9$ are $\frac{1}{10}$ of those in $54 \div 9$.

In general, multiplying a dividend and a divisor by the same number does not change the quotient. Multiplying by powers of 10 (e.g., 10, 100, 1,000, etc.) can be particularly useful for dividing decimals, as we will see in an upcoming lesson.

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Unit 5, Lesson 13
Dividing Decimals by Decimals

Let’s divide decimals by decimals.

13.1 Same Values

1. Use long division to find the value of $5.04 \div 7$.

2. Which of the following quotients has the same value as $5.04 \div 7$? Be prepared to explain how you know.
   a. $5.04 \div 70$
   b. $50.4 \div 70$
   c. $504{,}000 \div 700$
   d. $504{,}000 \div 700{,}000$

13.2 Placing Decimal Points in Quotients

1. Think of one or more ways to find $3 \div 0.12$. Show your reasoning.

2. Find $1.8 \div 0.004$. Show your reasoning. If you get stuck, think about what equivalent division expression you could write to help you divide.

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3. Diego said, "To divide decimals, we can start by moving the decimal point in both the dividend and divisor by the same number of places and in the same direction. Then we find the quotient of the resulting numbers."

   Do you agree with Diego's statement? Use the division expression $7.5 \div 1.25$ to support your answer.

Are you ready for more?

Can we create an equivalent division expression by multiplying both the dividend and divisor by a number that is *not* a multiple of 10 (for example: 4, 20, or $\frac{1}{2}$)? Would doing so produce the same quotient? Explain or show your reasoning.

13.3 Two Ways to Calculate Quotients of Decimals

1. Here are two calculations of $48.78 \div 9$. Work with your partner to answer the following questions.

```
      5.4 2
   9)4 8.7 8
   - 4 5
       3 7
     - 3 6
         1 8
       - 1 8
           0
```

Calculation A

```
                5.4  2
   9 0 0)4 8 7 8
       - 4 5 0 0
           3 7 8 0
         - 3 6 0 0
             1 8 0 0
           - 1 8 0 0
                 0   0
```

Calculation B

   a. How are the two calculations alike? How are they different?

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b. Look at Calculation A. Explain how you can tell that the 36 means “36 tenths” and the 18 means “18 hundredths.”

c. Look at Calculation B. What do the 3600 and 1800 mean?

d. We can think of $48.78 \div 9 = 5.42$ as saying “there are 9 groups of 5.42 in 48.78.” We can think of $4878 \div 900 = 5.42$ as saying “there are 900 groups of 5.42 in 4878.” How might we show that both statements are true?

2. a. Explain why $51.2 \div 6.4$ has the same value as $5.12 \div 0.64$.

b. Write a division expression that has the same value as $51.2 \div 6.4$ but is easier to use to find the value. Then, find the value using long division.

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13.4 Practicing Division with Decimals

Find each quotient using a method of your choice. Then discuss your calculations with your group and agree on the correct answers. If someone in your group makes an error, stop and help that person revise their work. If your group is unsure about an answer, consult your teacher.

1. $106.5 \div 3$
2. $58.8 \div 0.7$
3. $257.4 \div 1.1$
4. Mai is making friendship bracelets. Each bracelet is made from 24.3 cm of string. If she has 170.1 cm of string, how many bracelets can she make? Explain or show your reasoning.

Lesson 13 Summary

One way to find a quotient of two decimals is to multiply each decimal by a power of 10 so that both products are whole numbers.

If we multiply both decimals by the same power of 10, this does not change the value of the quotient. For example, the quotient $7.65 \div 1.2$ can be found by multiplying the two decimals by 10 (or by 100) and instead finding $76.5 \div 12$ or $765 \div 120$.

To calculate $765 \div 120$, which is equivalent to $76.5 \div 12$, we could use base-ten diagrams, partial quotients, or long division. Here is the calculation with long division:

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$$
\begin{array}{r}
6.375 \\
120 \overline{)\,765} \\
-\,720 \\
\hline
450 \\
-\,360 \\
\hline
900 \\
-\,840 \\
\hline
600 \\
-\,600 \\
\hline
0
\end{array}
$$

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Unit 5, Lesson 14
Using Operations on Decimals to Solve Problems

Let’s solve some problems using decimals.

14.1 Close Estimates

For each expression, choose the best estimate of its value.

1. $76.2 \div 15$
   - 0.5
   - 5
   - 50
2. $56.34 \div 48$
   - 1
   - 10
   - 100
3. $124.3 \div 20$
   - 6
   - 60
   - 600

14.2 Applying Division with Decimals

Your teacher will assign to you either Problem A or Problem B. Work together as a group to answer the questions. Be prepared to create a visual display to show your reasoning with the class.

Problem A: A piece of rope is 5.75 meters in length.

1. If it is cut into 20 equal pieces, how long will each piece be?
2. If it is cut into 0.05-meter pieces, how many pieces will there be?

Problem B: A tortoise travels 0.945 miles in 3.5 hours.

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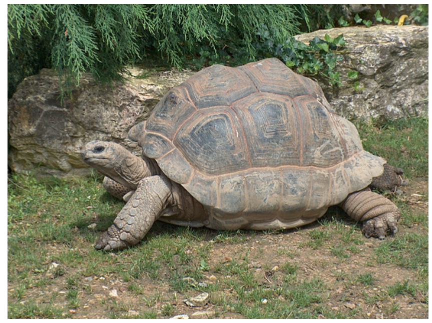
1. If it moves at a constant speed, how many miles per hour is it traveling?

2. At this rate, how long will it take the tortoise to travel 4.86 miles?

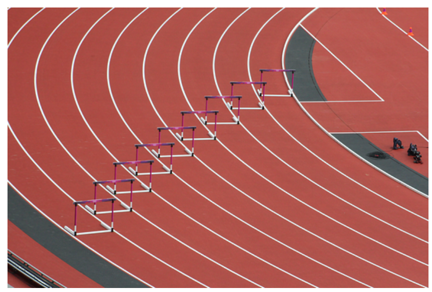


“Tortoise” by skeeze via Pixabay. Public Domain.

14.3 Distance between Hurdles

There are 10 equally-spaced hurdles on a race track. The first hurdle is 13.72 meters from the start line. The final hurdle is 14.02 meters from the finish line. The race track is 110 meters long.

1. Draw a diagram that shows the hurdles on the race track. Label all known measurements.



“August 3rd London Olympics 2012 stadium hurdles” by Steve Flair via Wikimedia Commons. CC BY 2.0.

2. How far are the hurdles from one another? Explain or show your reasoning.

3. A professional runner takes 3 strides between each pair of hurdles. The runner leaves the ground 2.2 meters *before* the hurdle and returns to the ground 1 meter *after* the hurdle.

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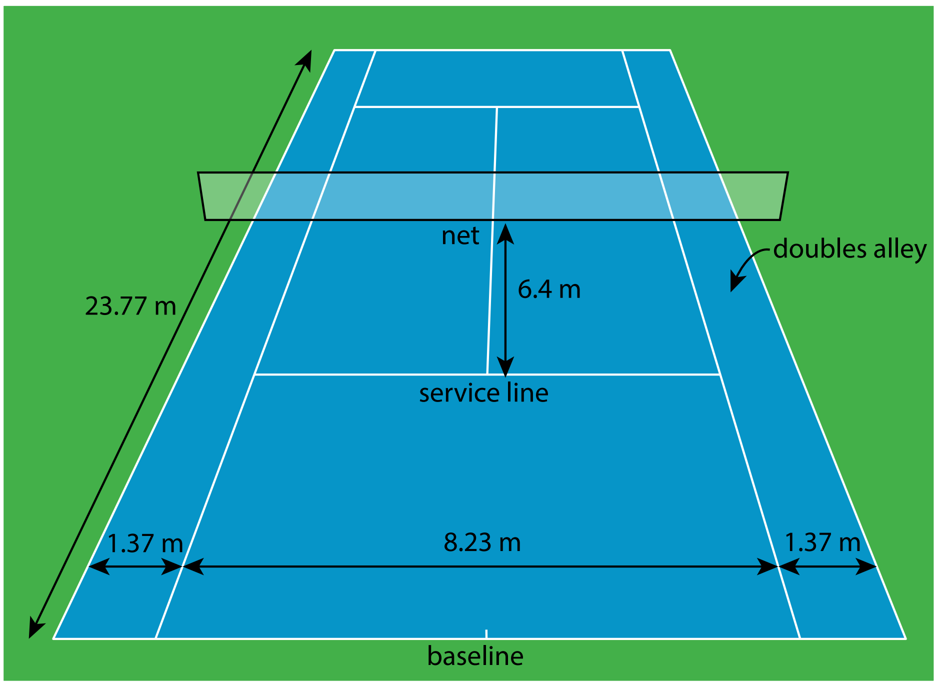
About how long are each of the runner's strides between the hurdles? Show your reasoning.

14.4 Examining a Tennis Court

Here is a diagram of a tennis court.



The full tennis court, used for doubles, is a rectangle. All of the angles made by the line segments in the diagram are right angles.

1. The net partitions the tennis court into two halves. Is each half a square? Explain your reasoning.

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2. Is the service line halfway between the net and the baseline? Explain your reasoning.

3. Lines painted on a tennis court are 5 cm wide. A painter made markings to show the length and width of the court, then painted the lines to the outside of the markings.

   a. Did the painter's mistake increase or decrease the overall size of the tennis court? Explain how you know.

   b. By how many square meters did the court's size change? Explain your reasoning.

Lesson 14 Summary

Diagrams can help us communicate and model mathematics. A clearly-labeled diagram helps us visualize what is happening in a problem and accurately communicate the information we need.

Sports offer great examples of how diagrams can help us solve problems. For example, to show the placement of the running hurdles in a diagram, we needed to know what the distances 13.72 and 14.02 meters tell us and the number of hurdles to draw. An accurate diagram not only helped us set up and solve the problem correctly, but also helped us see that there are only *nine* spaces between ten hurdles.

To communicate information clearly and solve problems correctly, it is also important to be precise in our measurements and calculations, especially when they involve decimals.

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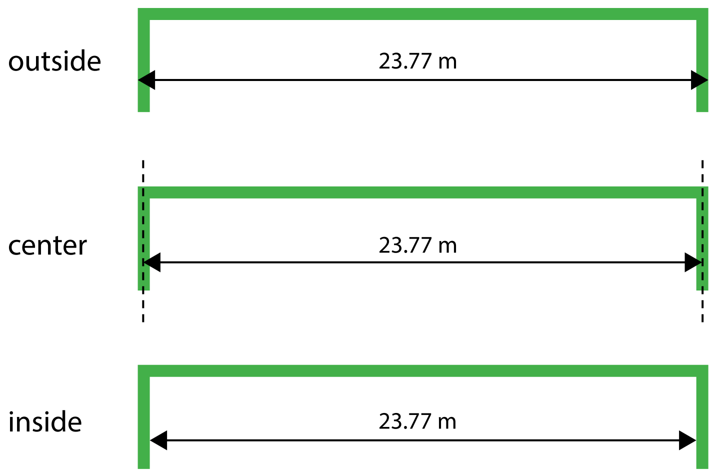
In tennis, for example, the length of the court is 23.77 meters. Because the boundary lines on a tennis court have a significant width, we would want to know whether this measurement is taken between the inside of the lines, the center of the lines, or the outside of the lines. Diagrams can help us attend to this detail, as shown here.



The accuracy of this measurement matters to the tennis players who use the court, so it matters to those who paint the boundaries as well. The tennis players practice their shots to be on or within certain lines. If the tennis court on which they play is not precisely measured, their shots may not land as intended in relation to the boundaries. Court painters usually need to be sure their measurements are accurate to within $\frac{1}{100}$ of a meter or one centimeter.

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Unit 5, Lesson 15
Making and Measuring Boxes

Let's use what we know about decimals to make and measure boxes.

15.1 Folding Paper Boxes

Your teacher will demonstrate how to make an open-top box by folding a sheet of paper. Your group will receive 3 or more sheets of square paper. Each person in your group will make 1 box. Before you begin folding:

1. Record the side lengths of your papers, from the smallest to the largest.
   - Paper for Box 1: ______________ cm
   - Paper for Box 2: _______________ cm
   - Paper for Box 3: ________________ cm
2. Compare the side lengths of the square sheets of paper. Be prepared to explain how you know.
   a. The side length of the paper for Box 2 is ________ times the side length of the paper for Box 1.
   b. The side length of the paper for Box 3 is ________ times the side length of the paper for Box 1.

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3. Make some predictions about the measurements of the three boxes your group will make:

   - The surface area of Box 3 will be ________ as large as that of Box 1.
   - Box 2 will be ________ times as tall as Box 1.
   - Box 3 will be ________ times as tall as Box 1.

Now you are ready to fold your paper into a box!

15.2 Sizing Up Paper Boxes

Now that you have made your boxes, you will measure them and check your predictions about how their heights and surface areas compare.

1. a. Measure the length and height of each box to the nearest tenth of a centimeter. Record the measurements in the table.

	side length of paper (cm)	**length of box (cm)**	**height of box (cm)**	**surface area (sq cm)**
Box 1				
Box 2				
Box 3				

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b. Calculate the surface area of each box. Show your reasoning and decide on an appropriate level of precision for describing the surface area (Is it the nearest 10 square centimeters, nearest square centimeter, or something else?). Record your answers in the table.

2. To see how many times as large one measurement is when compared to another, we can compute their quotient. Divide each measurement of Box 2 by the corresponding measurement for Box 1 to complete the following statements.

   a. The length of Box 2 is ________ times the length of Box 1.

   b. The height of Box 2 is ________ times the height of Box 1.

   c. The surface area of Box 2 is ________ times the surface area of Box 1.

3. Find out how the dimensions of Box 3 compare to those of Box 1 by computing quotients of their lengths, heights, and surface areas. Show your reasoning.

   a. The length of Box 3 is ________ times the length of Box 1.

   b. The height of Box 3 is ________ times the height of Box 1.

   c. The surface area of Box 3 is ________ times the surface area of Box 1.

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4. Record your results in the table.

	side length of paper	length of box	height of box	surface area
Box 2 compared to Box 1				
Box 3 compared to Box 1				

5. Earlier, in the first activity, you made predictions about how the heights and surface areas of the two larger boxes would compare to those of the smallest box. Discuss with your group:
   - How accurate were your predictions? Were they close to the results you found by performing calculations?
   - Let's say you had another piece of square paper to make Box 4. If the side length of this paper is 4 times the side length of the paper for Box 1, predict how the length, height, and surface area of Box 4 would compare to those of Box 1. How did you make your prediction?